

## **REAL-TIME OPTIMAL DEMAND RESPONSE FOR FREQUENCY REGULATION IN SMART $\mu$ GRID ENVIRONMENT**

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### **ABSTRACT**

Real-time demand response (DR) in smart  $\mu$ grid has been shown to be an effective tool for frequency regulation with increased penetration of renewable energy resources into the grid. Since DR is recognized as an incentive or direct payment to the participants, it is consequently desired to minimize the cost of DR for the utility. This paper presents an optimal DR strategy for minimizing the cost of DR for the utility in smart grid era. The economic model developed by Pennsylvania/New Jersey/Maryland (PJM) utility in the USA is used on an IEEE 13-bus standard system. Simulation results verify the effectiveness of the proposed approach to minimize the cost of DR for the utility. It is also shown that the DR, with or without optimization, decreases the overall cost of frequency regulation for the utility compared to the conventional spinning reserve, without sacrificing system stability.

### **KEY WORDS**

Adaptive hill climbing, ancillary services, demand side management, frequency regulation, locational marginal price, real-time demand response, spinning reserve.

## **1. Introduction**

Traditionally, frequency and voltage regulation, known as ancillary services, are provided by operating reserves which are basically flexible capacity generators, available when needed, to maintain secure operation of power systems. From an economic perspective, the above response services and reserve power are costly and any method which manages to reduce the magnitude of these services, without sacrificing system stability, is of significant importance [1] and [2].

Demand response (DR) traditionally refers to the changes in electricity usage by end-use customers from their normal consumption through direct control or interruptible/curtailable programs during peak hours [3]. Recently, this understanding of DR has been reformed because of the rapid progress in communication protocols and technology, where two-way communication with individual loads has become a reality. As a result, DR is redefined as the capability to aggregate and precisely control individual loads on command at all times [4].

One technical aspect of DR is providing ancillary services in a smart grid or  $\mu$ grid with high penetration of renewable power generation. It has been shown that real-time DR is effective in providing voltage and frequency regulation in a  $\mu$ grid in the absence of conventional voltage and frequency regulation services [5]-[13]. It is also well-known that there is a cost associated with DR. Utilities will need to provide an incentive or direct payment to encourage their customers to participate in their DR program. Thus, it is important for the utility to minimize this cost based on the available responsive loads and the different cost of DR. Since the ancillary services (i.e., frequency regulation) perform in a fraction of a minute (usually on the order of a few seconds), any optimization technique used for minimizing the cost of the ancillary services must be faster than the response time of the ancillary services.

While many studies have been reported on the different aspects of DR (including analyzing the potential of DR for ancillary services [5]-[13], planning and modeling [14]-[17], and the economic costs/benefit analysis of DR for the utility and customers [18]-[19]), there is no comprehensive study on real-time cost minimization of DR. This paper presents a comprehensive algorithm to minimize the cost of DR for the utility. A real-time DR algorithm developed by the authors in [12] and [13] and a DR model introduced by the Pennsylvania/New Jersey/Maryland (PJM) utility [16] are used. Because of the linearity of the objective function and the relatively small size of the problem, linear programming –simplex method– is used to solve the optimization problem. Results from the proposed algorithm are compared with those from the conventional ancillary services (i.e., spinning reserve) and the DR without optimization for further evaluations. The results of the study verify the effectiveness of the proposed optimal DR strategy to provide frequency regulation with reduction in cost for the utility, without sacrificing system stability.

The remainder of the paper is organized as follows: Section 2 presents the economic model of DR which is adopted from PJM real-time electricity market. The proposed optimal DR strategy is explained in Section 3. The system of study and its characteristics are introduced in Section 4. Simulation setups and results are discussed in Section 5. Finally, conclusion remarks are given in Section 6.

## 2. Economic Model of DR in PJM Electricity Market

Different economic models for DR have been proposed in the literature [14]-[18]. One of these models, which has been implemented in an actual power system, is introduced in [18]. PJM, as a regional transmission organization (RTO) and a part of the eastern U.S. interconnection grid with more than 51 million customers [20], established hourly markets for regulation and the contingency reserves including a real-time DR market. Two types of DR programs are offered by PJM: Emergency DR Program and Economic DR Program [18]. This section focuses on the latter, which is formalized as follows:

*PJM pays the Locational Marginal Price (LMP) to customers if the LMP in a given zone is above a trigger point (set by PJM at \$75/MWh). When the LMP is less than or equal to \$75/MWh, PJM pays the customer the difference between the LMP and the generation and transmission (G&T) components of the customer's bill [18]. In PJM, LMP is defined as the cost to serve the next MW of load at a specific location, using the lowest production cost of all available generation, while observing all transmission limits. The cost of marginal losses is not currently included in the PJM implementation of LMP [20].*

PJM offers this economic DR program in both its day-ahead and real-time markets. Although the PJM real-time economic DR program deals with hourly market, this model is adopted in instantaneous DR market in this study. The incentives to the customers who participated in the DR market will be in the form of payment related to the LMP at the time of the demand curtailment [18]. The direct payment to the  $i^{th}$  market participant curtailing 1 MW of demand (known as Cost of DR, CoDR) is given by [18]:

$$CoDR_{i,t} = \begin{cases} LMP_{i,t} & LMP_{i,t} \geq LMP^* \\ (LMP_{i,t} - GT_i) & GT_i < LMP_{i,t} < LMP^* \\ 0 & GT_i > LMP_{i,t} \end{cases} \quad (1)$$

where  $CoDR_{i,t}$  is the cost of DR for the participants in zone  $i$  at time  $t$ ,  $LMP_{i,t}$  is the locational marginal price in zone  $i$  at time  $t$ ,  $GT_i$  is the cost of generation and transmission for the participants in zone  $i$ , and  $LMP^*$  is the trigger locational marginal price.

Based on equation (1), if the  $LMP_{i,t}$  price is greater than the trigger price, then the utility will pay the full price to the DR participants at that time. Considering the fact that the  $LMP_{i,t}$  is basically the cost of the spinning reserve in zone  $i$  at time  $t$ , it seems as there is no benefit for the utility in this situation for doing DR. The truth is, as it is shown in the results section, the utility will save money even in this case because of loss reduction due to DR. In addition, there are certain economic advantages

for the utility in two other situations [the last two conditions given in equation (1)] due to the difference between the cost of spinning reserve and DR. Therefore, the utility will benefit through ancillary services provided by DR.

Once the  $LMP_{i,t}$  goes below the trigger price, the utility will not pay anything to the DR participants. Therefore, there is no motivation for the customers to participate in the real-time DR program. In this study, it is assumed that no individual loads participate in the real-time DR market when  $LMP_{i,t}$  is less than the trigger price. Consequently, two possible scenarios can be derived based on the  $LMP_{i,t}$  and  $LMP^*$ .

## 3. Proposed Optimal DR Strategy

In this study, the aforementioned PJM economic DR model is employed to develop the proposed optimal DR strategy. The goal is to minimize the cost of DR for the utility in the real-time market without sacrificing system stability. The schematic diagram of the proposed optimal DR strategy is depicted in figure 1. This strategy can be applied to any distribution feeder configuration with the three following assumptions:

- I. Two-way communication is required to the individual customers' loads.
- II. The size of the system will determine the complexity of the optimization problem. Therefore, achieving optimal or near optimal solution should be feasible in a given time step.
- III. The system frequency is assumed to be the same at different locations along the grid, which is a reasonable assumption for small distribution feeders. Although, the proposed method can be extended for larger distribution feeders with different frequency.

As shown in figure 1, the single frequency measurement at the point of common coupling (PCC) is the input to the "DR Strategy Unit". This unit is in charge of determining the total amount of responsive load which has to be turned off to balance generation and demand and stabilize the system frequency regardless of its interruption cost.

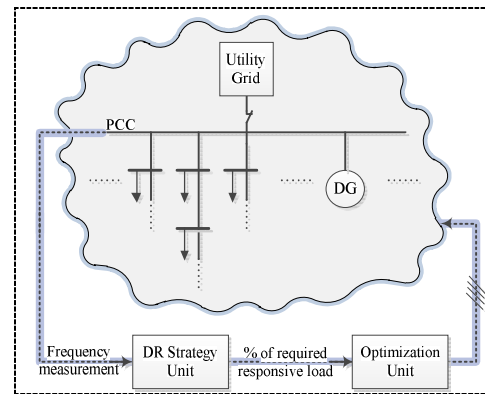


Figure 1. Schematic diagram of the proposed optimal DR strategy

Two DR strategies have been developed by the authors. One is based on adaptive hill climbing (AHC) control [12] and the other, called Comprehensive Controller, is based on the combination of the AHC and a step-by-step (SBS) controller [13]. The comprehensive control strategy discussed in [13] is used in this study. In this control strategy, the AHC controller calculates the required amount of responsive load to be turned off depending on the frequency deviation from its normal value, and the SBS controller takes action once the AHC controller has brought the frequency deviation within  $\pm 0.05$  Hz. These control strategies are explained in section 3.1 and 3.2.

### 3.1 AHC strategy

During normal operation, the frequency is assumed to be within the desired dead-band  $\Delta f = f_{desired} \pm 0.05$  Hz. Under a disturbance, the system frequency will increase or decrease depending on the type of disturbance. When the frequency deviation goes out of the pre-defined dead-band, the AHC controller will start to regulate the frequency by changing the amount of responsive loads [12] and [13]. The frequency, measured at the PCC of the microgrid, is the input variable to the controller. If the frequency deviation falls outside the dead-band, a percentage of the dynamic load will change based on the sign and magnitude of the frequency deviation as follows:

$$\%Load(k) = \%Load(k-1) + \Delta f \times M \quad (2)$$

Based on equation (2), when the frequency is higher than acceptable, a percentage of the responsive loads (that are off) will turn on, and when it is lower than acceptable, a percentage of the responsive loads (that are on) will turn off. The perturbation parameter is  $M \times \Delta f$ , where  $M$  is a constant, is used to scale down the frequency deviation. More detail on the AHC algorithm is provided in [12].

### 3.2 SBS Controller

In order to assure that the minimum required amount of responsive load is changed, the SBS controller is used. Once the frequency is stabilized by the AHC controller, the step-by-step controller will start operating to minimize the amount of manipulated responsive loads. Using the SBS controller, the manipulated responsive load will be decreased by 5% at each one-second time-step according to equation (3) until the frequency exceeds the desired dead-band.

$$\%Load(k) = 0.95 \times \%Load(k-1) \quad (3)$$

According to equation (3), the responsive load variation depends on its previous value. Therefore, the load control strategy begins with large variations in load which decrease with time, making the “step-by-step”

control strategy non-linear with large variations at the beginning and small changes at the end. As a result, the proposed control strategy minimizes the amount of responsive loads that need to be manipulated at steady-state to keep the system frequency within the desired range.

The SBS controller is responsible for minimizing the amount of manipulated responsive load at steady-state and therefore provides improved quality of service (QoS) to the customers. As a result of improved customers' QoS, a larger amount of responsive load will be available for further frequency stabilization in case needed. This way, the cost of DR to the utility is reduced because fewer customer loads are manipulated.

Once the total amount of manipulated load has been determined by the “DR Strategy Unit”, the “Optimization Unit” calculates the share of each individual load in the DR market based on the final cost for the utility. The objective function is set to minimize the cost of DR for the utility subject to some equality and inequality constraints. Different heuristic and gradient-based optimization techniques can be used in this study based on two important factors: the time interval of optimization and the size of the problem. Usually, gradient-based methods have very slow convergence and high probability of being trapped in the local minima. However, they are more reliable for near optimal solutions in larger systems. In this study, as a first step, linear programming (LP) - simplex method is chosen for cost optimization because of the linear nature of the objective function and linear equality/inequality constraints. MATLAB/Optimization Toolbox [21] has been used to evaluate the effectiveness of the method in real-time cost optimization for DR. The application and evaluation of other classical and heuristic optimization techniques are a part of the authors' future work.

### 3.3 LP-Simplex Method

LP has a diverse range of real-life applications in economic analysis, planning, operations research, computer science, and engineering due to its simplicity [22]. It is well-known that the number of iterations in the simplex method is just a small multiple of the problem dimension [22], which consequently hold it as a promising candidate in this study. The general form of LP problem is stated as [22]:

$$\begin{aligned} \min f(x) &= C^T \cdot x \\ \text{Subject to : } &\begin{cases} A \cdot x = b \\ x \geq 0 \end{cases} \end{aligned} \quad (4)$$

where  $f(x)$  is the objective function,  $x$  is the decision variable vector,  $C$  is the linear objective function coefficient vector, and  $A$  and  $b$  are the equality constraints. Further details on the LP-Simplex method can be found in standard optimization textbooks, e.g. [22].

According to equation (1), two different scenarios can be designed based on the  $LMP_{i,t}$ :

SCENARIO I: When the trigger price ( $LMP^*$ ) is assumed to be less than  $LMP_{i,t}$ . In this case, based on equation (1), the utility is responsible to pay the fixed  $LMP_{i,t}$  price to the DR market participants. Therefore, the price for DR for all individual loads is the same as the cost of spinning reserve, and there is no need for optimization based on the final cost of DR for the utility. However, a loss minimization algorithm for DR will help to show a possible increase in the benefits of DR for the utility. This cost of DR (CoDR) for the utility can be calculated as follows:

$$CoDR_t = \sum_i (P_{i,t}) \times LMP_{i,t} \quad (5)$$

*for each period*

where  $P_i$  is the amount of manipulated power from responsive load  $i$  (MW),  $t$  is the time duration (hour), and  $i$  is the number of responsive loads participating in the DR market.

SCENARIO II: In this scenario,  $GT_i < LMP_{i,t} < LMP^*$ . In this case, according to equation (1), the DR participants will receive the difference between  $LMP_{i,t}$  and  $GT_i$  cost in their zone. Since the price of generation and transmission for different individual loads is not the same, the cost of DR for the utility will be different for different customers. Therefore, it is beneficial to the utility to do DR based on the minimum cost, and based on equation (4), the LP problem for the proposed optimal DR can be defined as

follows:

$$\begin{aligned} \min CoDR_t &= \sum_i (P_{i,t}) \times (LMP_{i,t} - GT_i) \\ \text{Subject to : } &\begin{cases} \sum_i P_i = P_{total,t} \\ P_i < P_{i,max} \end{cases} \end{aligned} \quad (6)$$

where  $P_{total,t}$  is the total amount of responsive load which should be turned off, and  $P_{i,max}$  is the maximum amount of responsive load  $i$  participating in the DR market. Remember that  $P_{total,t}$  is already determined by the “DR Strategy Unit” at each time step.

#### 4. System of Study

To verify the effectiveness of the proposed optimal DR strategy, it was implemented on the IEEE standard 13-bus distribution network, [23] and [24], shown in figure 2.

In order to be able to observe the frequency behavior of the system as a microgrid, the utility source of the original model was replaced with a 15-MW diesel generator (DG). The torque of the DG can be adjusted to set the DG’s operating point (e.g., to light or heavy loading).

The dynamic model for the diesel engine with a speed governor and excitation controller and synchronous generator are extracted from MATLAB/Simulink SimPowerSystems toolbox [25]. The parameters of the DG are given in Table 1 [25].

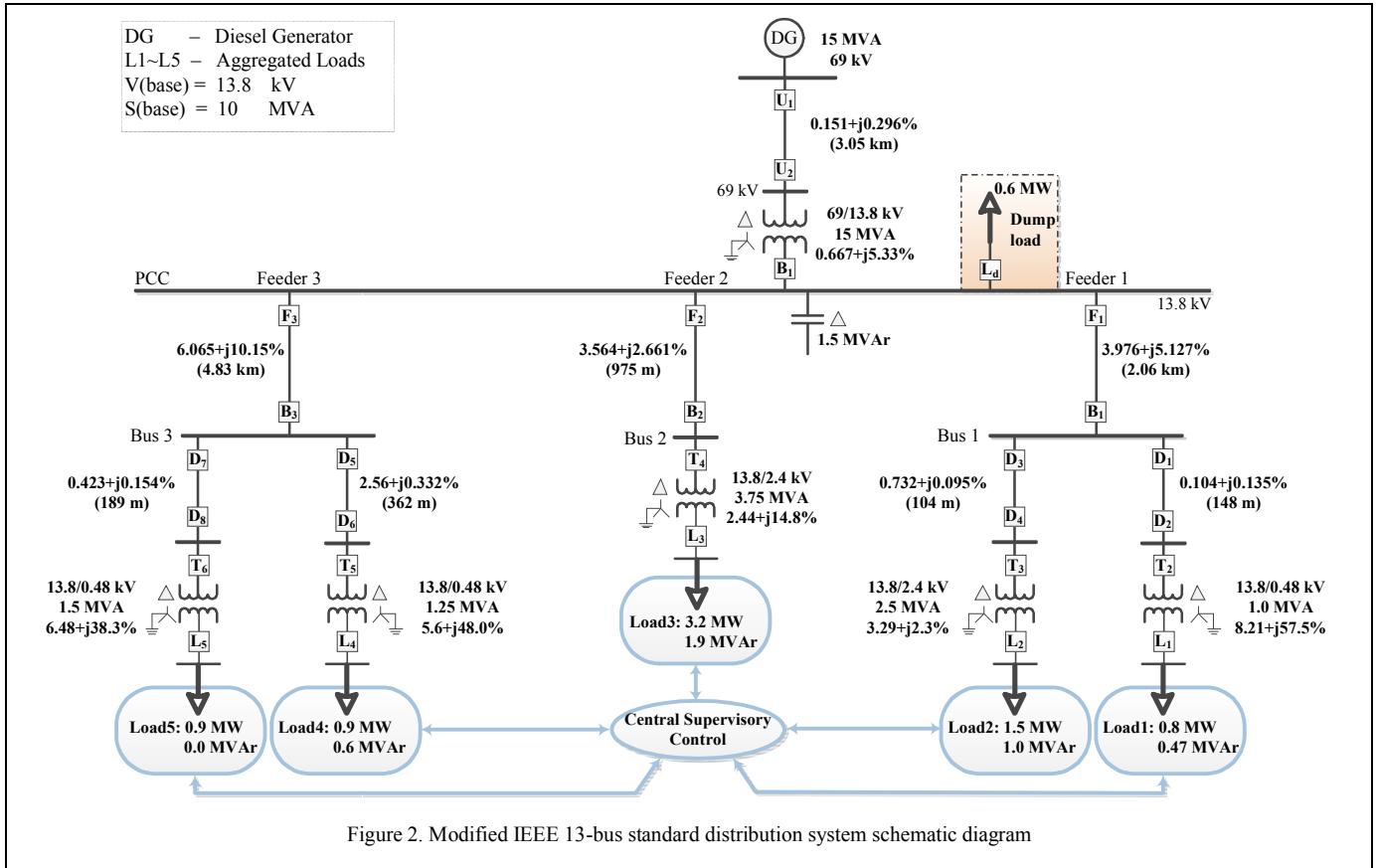


Figure 2. Modified IEEE 13-bus standard distribution system schematic diagram

Table 1  
Diesel generator parameters

Synchronous generator parameters	
Nominal power	15 MVA
Line-to-line voltage	69 kV
Nominal frequency	60 Hz
d-axis reactance ( $X_d, X_d', X_d''$ )	1.305, 0.296, 0.252 p.u.
q-axis reactance ( $X_q, X_q', X_q''$ )	0.474, 0.243, 0.18 p.u.
Inertia coefficient	2.2 sec
Governor and Diesel Engine parameters	
Regulator gain	40
Regulator time constants ( $T_1, T_2, T_3$ )	0.01, 0.02, 0.2 sec
Engine time delay	0.14 sec
Excitation controller parameters	
Regulator gain	200
Regulator time constants	0.02 sec
Damping filter gain and time constant	0.001, 0.1 sec

To observe the system frequency deviation, the maximum mechanical torque of the diesel generator is limited to a certain point in each case. A 0.6-MW load is also added at the PCC to increase the overall load so that the frequency deviation from 60 Hz can be noticeable.

The distribution system shown in figure 2 includes five aggregated loads which are located in different zones of the feeder with different losses. 15% of the nominal capacity of each load is considered to be available for DR. Also, two-way communication between the utility control center and each load is assumed to be available. The GT cost for the different loads maybe different, depending on their location. The GT costs used in this study are given in Table 2. The  $LMP_{i,t}$  is assumed to be the same for all five zones, as  $LMP_t$ .

Table 2  
G&T prices for different zones

	Load 1 ( $i=1$ )	Load 2 ( $i=2$ )	Load 3 ( $i=3$ )	Load 4 ( $i=4$ )	Load 5 ( $i=5$ )
GT <sub>i</sub> (\$/MWh)	47	45	40	53	50

## 5. Simulation Studies

As mentioned earlier, two different scenarios are considered in this study for DR and conventional frequency regulation cost calculations. As discussed in section 3.3, SCENARIO I, the cost of DR for all individual loads is the same as the cost of spinning reserve. Therefore, no cost optimization is needed for this scenario. In this case, simulation studies have been carried out for conventional frequency regulation (Con-f-reg) and DR with no optimization (DR-no-opt). However, SCENARIO II includes three different studies: “Con-f-reg,” “DR-no-opt,” and Optimal DR (Opt-DR). In the case of “Con-f-reg,” spinning reserve is considered as conventional ancillary services which will be provided by the diesel generator. Therefore, the limitations on the diesel generator torque have been removed in this case. In other words, 100% of the load demand is accommodated by the diesel generator with the price of  $LMP_t$ . Furthermore, the cost/benefit analysis is only performed for a heavy loading condition, under which the frequency

drops below 60 Hz and the utility needs to disconnect a part of the load if spinning reserve is not used.

All calculations have been done for 50 sec, the DR strategy is updated every 20 msec, and the average computation time of optimization is less than 1 msec. All simulations are done on a desktop with 2 Intel® Xeon® 2.13 GHz processors.

### 5.1 SCENARIO I:

In this case, the 0.6-MW dump load is added to the  $\mu$ grid at  $t=7$  sec. The locational marginal price ( $LMP_t$ ) and the trigger price ( $LMP^*$ ) are assumed to be \$90/MWh and \$75/MWh, respectively. These are the prices used by PJM [18]. As a result, system frequency starts to deviate from 60 Hz. In the “Con-f-reg” case, the excess required power is provided by the diesel generator while in the “DR-no-opt” case the balance between generation and demand is achieved through DR. As a result, the “DR-no-opt” case shows 1.98% reduction in the cost of frequency regulation through DR for the utility compared to the “Con-f-reg” case through purchasing more power from spinning reserve. This percentage of improvement, especially when the cost of spinning reserve and DR are the same, is a consequence of less power loss in the  $\mu$ grid as a result of DR. It is well-known that power loss reduction is one of the countless advantages of DR for the utility through disconnecting a portion of the responsive loads. The 1.98% savings proves this fact in this relatively small  $\mu$ grid.

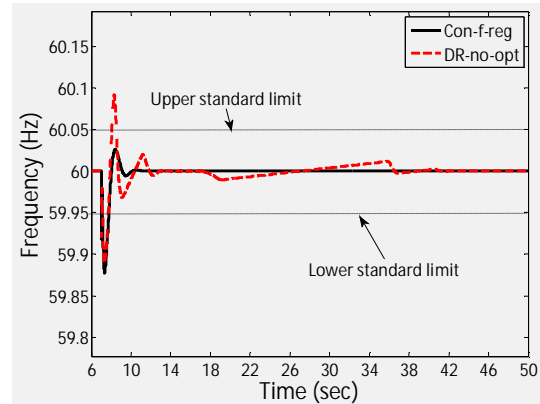


Figure 3. System frequency in SCENARIO I

Figure 3 shows the system frequency for the two cases in SCENARIO I. It can be observed that the frequency is in the acceptable range ( $60 \pm 0.05$  Hz) in both cases studied. But, as noted above, the “DR-no-opt” case achieves frequency stabilization with a reduced cost of frequency regulation for the utility.

In figure 4, the percent variation in responsive load is shown for the “DR-no-opt” case. During the transient period, more responsive loads are used to regulate the system frequency as fast as possible. However, the overall responsive load variation was not jeopardized the system stability. The negative sign by the responsive load changes in figure 4 shows a decrease in the amount of



responsive load (i.e., the percentage of responsive loads that have been turned off at any given time). In the “Con-f-reg”, no responsive load has been changed. In other words, frequency regulation has been provided by the diesel generator in the “Con-f-reg” case.

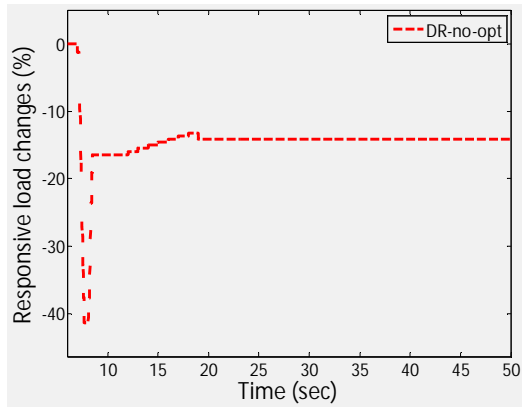


Figure 4. Total responsive load changes in SCENARIO I

## 5.2 SCENARIO II:

In this scenario, in addition to the “Con-f-reg” and the “DR-no-opt”, the proposed optimal DR is also applied. The locational marginal price (LMP<sub>i</sub>) and the trigger price (LMP<sup>\*</sup>) are assumed to be \$60/MWh and \$75/MWh, respectively. The same simulation studies, carried out in SCENARIO I, were also carried out for scenario II for all three cases. Both the DR strategies show a significant reduction in the frequency regulation cost. The “DR-no-opt” case has 23.8% cost reduction and the “Opt-DR” has a cost savings of 28.8%. Therefore, the cost savings of the “Opt-DR” compared to the “DR-no-opt” is 5% in this case. The cost savings through “Opt-DR” will be more significant for larger grids and  $\mu$ grids.

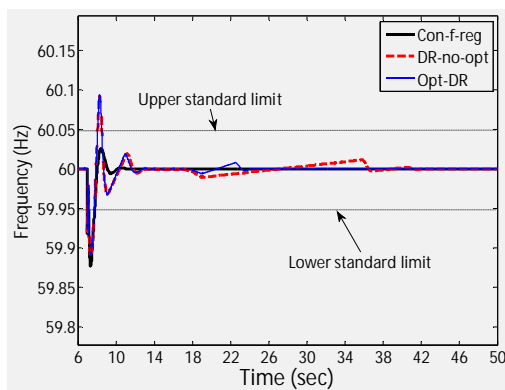


Figure 5. System frequency in SCENARIO II

Figure 5 shows the frequency response for the three cases discussed. It can be concluded that the DR strategy has no significant negative impact on the system frequency regulation compared to conventional ancillary services. The small spike in frequency during the transient period for the DR strategy has negligible effect on the system frequency response.

In figure 6, the response of DR strategy for responsive load variation is shown for both the “DR-no-opt” and “Opt-DR”. The amount of manipulated responsive load is essentially the same in both cases.

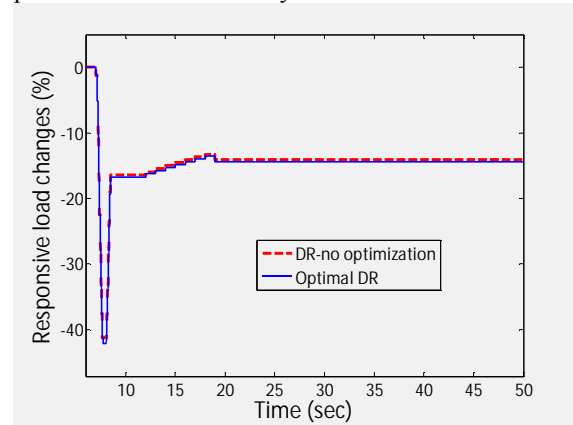


Figure 6. Total responsive load changes in SCENARIO II

## 6. Conclusion

This paper presents a general optimal DR strategy for frequency regulation in a  $\mu$ grid. The proposed algorithm includes a comprehensive DR strategy, presented in [13], along with the optimization technique (LP-simplex method). Simulation results show the effectiveness of the general DR strategy versus conventional ancillary services through spinning reserve. More importantly, the proposed optimal DR strategy shows even more cost reduction for the utility while it shows no significant negative impact on the frequency response.

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