

Consumers' Flexibility Estimation at the TSO Level for Balancing Services

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Abstract—Demand flexibility will be an inevitable part of the future power system operation to compensate stochastic variations of ever-increasing renewable generation. One way to achieve demand flexibility is to provide time-varying prices to customers at the edge of the grid. However, appropriate models are needed to estimate the potential flexibility of different types of consumers for day-ahead and real-time ancillary services (AS) provision. The proposed method should account for rebound effect and variability of the customers' reaction to the price signals. In this study, an efficient algorithm is developed for consumers' flexibility estimation by the transmission system operator (TSO) based on offline data. No aggregator or real-time communication is involved in the process of flexibility estimation, although real-time communication channels are needed to broadcast price signals to the end-users. Also, the consumers' elasticity and technical differences between various types of loads are taken into account in the formulation. The problem is formulated as a mixed-integer linear programming (MILP) problem, which is then converted to a chance-constrained programming to account for the stochastic behaviour of the consumers. Simulation results show the applicability of the proposed method for the provision of AS from consumers at the TSO level.

Index Terms—Rational end-users, transmission system operator, flexibility resources, ancillary services, chance-constrained programming

NOMENCLATURE

A. Sets:

- T Set of time, indexed by t , $t \in [1, \dots, \tau]$.
- J Set of end-users' categories, indexed by j .
- α Type of regulation, i.e., up- or down-regulation.

B. Parameters:

- λ^{base} Baseline electricity price [DKK cent/kWh].
- $\Delta\lambda_t^\alpha$ Time-varying electricity price (called delta price) for regulation type α at time t [DKK cent/kWh].
- $\underline{\Delta\lambda}_j^\alpha, \overline{\Delta\lambda}_j^\alpha$ Minimum and maximum delta prices for regulation type α of end-users' category j [DKK cent].
- $L_{t,j}^{\text{base}}$ Baseline end-users' demand of category j at time t [kW].
- $L_{t,j}^{\min}, L_{t,j}^{\max}$ Minimum and maximum electricity consumption of end-users' category j at time t [kW].
- $a_{t,j}^\alpha$ Actual willingness of end-users' category j to provide flexibility type α at time t [p.u.].
- \bar{a}_j^α Maximum willingness of end-users' category j to provide flexibility type α [p.u.].

- r_j^α Ramp-rate of end-users' category j for regulation type α [kW/h].
- n_j^α Maximum number of activation times for end-users' category j to provide flexibility type α .
- $\underline{d}_j^\alpha, \bar{d}_j^\alpha$ Minimum and maximum continuous flexibility duration of end-users' category j when activated to provide flexibility type α [h].
- β_{th} Theoretical confidence level imposed in the chance-constrained programming.
- β_{ac} Actual confidence level achieved in the chance-constrained programming.
- R_j Maximum rebound delay for end-users' category j [h].

C. Variables:

- $L_{t,j}^\alpha$ Flexibility of end-users' category j at time t for regulation type α [kW].
- $u_{t,j}^\alpha$ Binary variables, indicating flexibility status of end-users' category j at time t for regulation type α .
- $y_{t,j}^\alpha, z_{t,j}^\alpha$ Starting and stopping binary variables of end-users' category j at time t indicating flexibility type α .

I. INTRODUCTION

In recent decades, a significant amount of renewable energy sources (RES) has been integrated into power systems, supported by the increasing global awareness towards climate change and the tremendous cost reduction in the new technologies [1]. While offering unquestionable environmental benefits and sustainability in energy production, large penetration of RES introduces new concerns and challenges in power systems planning and operation because of an unprecedented level of stochasticity, non-linearity, and dynamics [2]. Consequently, it causes higher risk of frequency deviation, voltage excursion, and network congestion in real-time operation. Furthermore, it requires larger amount of ancillary services (AS) to compensate demand and generation imbalances in real-time. AS consist of a variety of operations, beyond the electricity generation and transmission. These operations guarantee service quality, continuity and security from distribution (e.g., voltage regulation) to transmission level (e.g., frequency regulation and congestion management). Since RES are located at different levels of the grid, challenges are extended to all aspects of AS provision. This further demands a holistic change in AS provision in the future power system with high RES penetration.

An attempt of that nature is the so-called demand response (DR) programs. Different types of DR programs have been

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developed and tested in the last decade or so [3]. These include the application of time-of-use (ToU) rates, incentives, real-time prices (RTP) and direct load control (DLC). ToU schemes define different rates at different time of the day (i.e., usually two-tiered peak and off peak [4]) but that do not change based on the condition of power system. Incentives are designed to be added on top of a flat electricity retail price. The consumer is always rewarded to alter its consumption to support the DR scheme voluntarily. However, they are used in relation to two-way communication schemes [5]–[7]. Finally, RTP are generated to reflect the real-time condition of the grid [8]. RTP is different from incentives, as in RTP consumers only receive a time-varying price. On the other hand, in incentive-based schemes, consumers still receive a flat retail price and, on top of that, they can agree on an incentive to alter their consumption. This solution preserves consumers' autonomy as it is based on one-way communication structure. Prices are broadcast to consumers which autonomously decide how to respond to them through decentralised controllers. Also, no control signal is submitted to the consumers, and the same price signal can be broadcast to a various pool of consumers (i.e., at their HEMSs), as its formulation is not device-based.

Such price schemes have been used in the Olympic Peninsula Demonstration project, where the procurement of demand flexibility in response to 5-minute price signals was successfully tested [9]. Although RTP might potentially increase price volatility, it is possible to address such a concern by properly designing the price, e.g., imposing a fixed price cap [10]. The RTP can also be agreed in a market-based approach, such as in transactive energy (TE) [11]. TE allows the consumers to be actively involved in the formation of the price, which in turn reduces uncertainty in consumers' response. However, this type of methods requires regular feedback from the consumers for flexibility estimation, requiring costly and cyber security-prone two-way communication infrastructure.

Another type of DR programs is centralised and decentralised DLC schemes [12]. In centralised DLC mechanisms, an external entity directly controls consumers' load through a two-way communication link [13]. Although such solutions substantially reduce uncertainty in the consumers' response [14], they compromise consumers' privacy and autonomy [11]. In fact, consumers have to allow an external entity to decide about the way they consume electricity. In [15] and [16], it is shown that consumers might be reluctant in losing control of their consumption, and that automation of the consumption is accepted only if consumers can autonomously manage it. To gain higher acceptance from consumers towards DLC mechanisms, long-term contracts [17] have also been formulated. The main challenge of such approaches is that consumers need to plan their future consumption ahead of time, which most of the consumers are not accustomed to do so [6]. Therefore, only part of the available flexibility might be exploited in such programs. An alternative to centralised DLC schemes is decentralised DLC, which uses one-way communication [18]. It is implemented by simply broadcasting a control signal from a centre, where the ultimate decision is made by the local controller at the consumer's side. This arrangement addresses privacy and comfort issues in the DLC

schemes (i.e., each distributed controller individually satisfies the consumer' constraints [19]). However, the control signal generated by the central controller is based on models for specific types of loads. Therefore, different specialised control signals should be issued for every type of loads in order to exploit the existing potential flexibility [20], [21]. In addition, the control signals are generated by assuming a linear model for the device, which might not represent the true dynamics of the underlying appliance, thus it might be error-prone. Nevertheless, it is true that the error might decrease as the number of aggregated devices grows.

While the authors acknowledge the benefits and disadvantages of various RTP and DLC methods, the RTP scheme is assumed in this study and the proposed flexibility estimation algorithm is developed based on the RTP concept. From the perspective of the transmission system operator (TSO), RTP must be properly formulated to address the desired aggregated change in consumption that solves the operational problems. Therefore, understanding how end-users respond to different price signals in an aggregated manner can help the TSO to estimate the potential of demand flexibility and design price signals accordingly [22]. In other words, by utilising appropriate models, the system operator can evaluate the impact of different prices on consumers' flexibility to determine the right price to obtain a certain amount of flexibility [23]. Unfortunately, literature scarcely reported load flexibility estimation from the system operator's point of view. In [24], a daily load response model for different end-users' categories is proposed based on the day-ahead spot market prices. However, the stochastic responsiveness of different end-users' categories and consumers' preferences have not been studied. Moreover, only few papers investigated the flexibility potential of various industrial loads [25], despite the fact that 80% of electricity usage is consumed in this sector in some countries [26]. Therefore, there is a gap in knowledge to properly estimate aggregated flexibility of the consumers while accounting for stochasticity in their elasticity and preferences without real-time communication links.

In this paper, an optimisation problem is formulated to estimate the aggregate flexibility of rational end-users (REUs) with different elasticity and preferences at the TSO level in response to time-varying prices. The proposed tool can be used to quantify the amount of demand flexibility that is available for balancing. Estimating the amount of load flexibility in response to different prices can be useful for an aggregator to build blocks of load capacity bids for different time intervals (e.g., hourly, in CAISO). Although how to generate the time-varying prices is out of the scope of this study, the proposed method can also be used to evaluate the impact of different prices on demand flexibility. Moreover, balancing requirements might change due to the prediction errors in the load demand and renewable generation and unexpected outages. Therefore, having an estimate of the available load flexibility can be very useful during the real-time operation of the power system. Within this context, our method can be used to provide such an estimate both in advance or in real-time. Furthermore, having more flexible resources (from generation and demand) enhances competition in the balancing

market, resulting in price reduction that ultimately reduces electricity prices for the end-users. In order to reduce the negative impact of the consumers' stochastic behaviour on the estimated flexibility, the original formulation is converted to a chance-constrained (CC) programming, where the risk level of the solutions can be guaranteed. The main contributions of the paper can be summarised as follows:

- Quantifying the aggregated up- and down- flexibility from various types of consumers' categories at the TSO level to address AS requirements;
- Formulating a chance-constrained optimisation to account for the stochasticity in the consumers' willingness in such an application;
- Developing a statistical model of aggregated consumers' willingness (i.e., elasticity and preferences) for different categories of consumers and incorporating it in the optimisation problem.

The rest of the paper is organised as follows: Section II presents the theoretical foundation for the formulation in terms of time-varying prices and REUs. It is followed by a deterministic optimisation formulation of the aggregated load flexibility in Section III. Then, the formulation is converted to a CC programming problem to address stochasticity of the end-users' behaviour in Section IV. In Section V, a case study is proposed and a series of simulations are carried out to show the effectiveness of the proposed model. Simulation results are discussed and the paper is finally concluded in Section VI.

II. MODELLING CONCEPTS

Quantifying demand flexibility at the TSO level with limited aggregated historical data inevitably involves complex parameters and conditions, which must be simplified for appropriate modelling. To keep the proposed method practical and computationally tractable, two important assumptions are made based on the current trend in smart grid technologies, as explained below.

A. Time-varying prices

Time-varying prices are assumed to exist to activate consumers' flexibility in this study. In the smart grid era, the application of advanced metering infrastructure will further support the time-varying pricing mechanism in practice. Without loss of generality and similar to the Olympic Peninsula Demonstration, it is assumed that time-varying prices are superimposed on the existing retail electricity price. We refer to the existing flat retail price as the "baseline price", λ^{base} , while the time-varying price component is called "delta price" in the rest of the paper. The latter is denoted by $\Delta\lambda_t^\alpha$, representing the time-varying price for flexibility type α at time t . Depending on the grid condition, upward regulation (i.e., $\alpha = u$) or downward regulation (i.e., $\alpha = d$) may be required. In the existing terminology, regulation is defined from the generators' perspective, e.g., in California ISO [27], where a load increase is equivalent to a decrease in generation (i.e., down-regulation) and vice versa. Therefore, down-regulation is achieved from negative delta prices, $\Delta\lambda_t^d$ or equivalently $\Delta\lambda_t^\alpha : (\Delta\lambda_t^\alpha < 0)$. On the other hand, load reduction is

equivalent to an increase in generation (i.e., up-regulation), which is achieved by positive delta prices, $\Delta\lambda_t^u$ or equivalently $\Delta\lambda_t^\alpha : (\Delta\lambda_t^\alpha > 0)$. Since the source of real-time operation issues can be linked to many entities (e.g., load, generation plants, transmission and distribution networks, interconnected areas, and so on), it would be unfair to the consumers to pay more because of the issues that were probably initiated by other stakeholders [23]. To alleviate such a problem, zero accumulated delta prices should be enforced at the end of each day:

$$\sum_{t=1}^{\tau} \Delta\lambda_t^u + \Delta\lambda_t^d = 0 \quad (1)$$

Summing the delta prices to zero over a day of operation is preferred in this paper instead of the alternative approach, which is the sum of the demand-weighted prices. The main reason is that it is difficult to predict the aggregated response of each consumers' category in the hours ahead, which leads to higher uncertainty in the demand-weighted prices. By providing delta-prices whose sum is zero, some periods of low prices are ensured to exist from which the consumers can benefit (i.e., by responding to the time-varying price). In the simulation study, it is assumed that the delta prices are known in advance by the TSO. In the electricity markets where energy and AS are procured simultaneously in the day-ahead market, e.g., California ISO [28], such AS prices are available. Furthermore, the proposed tool could be readily used for real-time operation in a rolling horizon fashion to incorporate potential updates of the prices and load flexibility provided in previous hours.

B. Rational end-users (REUs)

Since manual consumers' reaction to the price signal is not practical nor effective, energy management systems (EMS) are required to successfully implement price-based DR programs in practice. Once the time-varying price is received by the EMS, they run an individual optimisation and/or control problem locally to minimise the incurred electricity cost accounting for the customers' preferences [8], [29]–[31]. As an important smart grid technology, the EMS market value reached US\$4 billion in 2017 [32]. With the current market trend, it is likely that most of the future electricity consumers will have EMS at their premises. This, in turn, will enhance the elasticity of demand to time-varying prices, which is a key feature in successful DR implementation. In addition, application of EMSs improves the predictability of consumers' response to price signals while avoiding communication of any sensitive information over communication channels in real-time.

In this paper, we deal with EMS-equipped end-users, which are called REUs, to receive the time-varying electricity prices through one-way communication channels. The diversity of the REUs' behaviour towards the delta prices is modelled below.

1) *REUs' responsiveness to the price signal*: In order to appropriately model the diversity of consumers' flexibility, the willingness of each REU to deviate from its baseline demand, i.e., $\mathbf{L}_{t,j}^{\text{base}}$, is modelled as a stochastic phenomenon. Generally, the price-responsiveness of a consumer depends on various factors, e.g., weather conditions, electricity price,

time and type of day, and season, etc. [33]. As an example, in [34], it is shown that the response of the load demand has been faster in the cold weather. In this paper, however, only electricity price, type of consumers and time of the day are considered in the REUs' responsiveness modelling, i.e., $\mathbf{a}_{t,j}^\alpha$, to keep the problem tractable. Other factors, such as the weather condition and type of day, could be included in the current model by adjusting the willingness parameter $\mathbf{a}_{t,j}^\alpha$, e.g., as a function of the ambient temperature and type of day. Weather conditions are neglected because various types of end-users react differently to the weather conditions. Therefore, proper data is needed to estimate the relationship, which is not available to the public at the moment. The value of $\mathbf{a}_{t,j}^\alpha$ varies within the range of $[-1, 1]$, where 0 indicates no intention to change consumption and 1 (-1) represents a 100%-increase (decrease) in consumption in response to the delta price. From literature, [24] approached the consumers' price-responsiveness in a similar manner to investigate the behaviour of a pool of end-users. Consumers' willingness, however, was considered constant over time and price in that study. Halvgaard et al. in [35] adopted a linear model of price and consumption to formulate the price response behaviour. In [36], Aalami et al. focused on nonlinear functions, which better describe the price response behaviour compared to the linear models. Following the work of Aalami, we adopted a power function to model the consumers' willingness, as shown in Fig. 1. Similar to [24] and [37], where the authors assumed a price threshold for achieving DR, a dead-band is considered to address the fact that consumers become responsive beyond a certain price. Therefore, for a delta price smaller than the dead-band price, i.e., $\underline{\Delta\lambda}_j^\alpha$ in the specific regulation direction, no response is expected from the pool and the flexibility is zero:

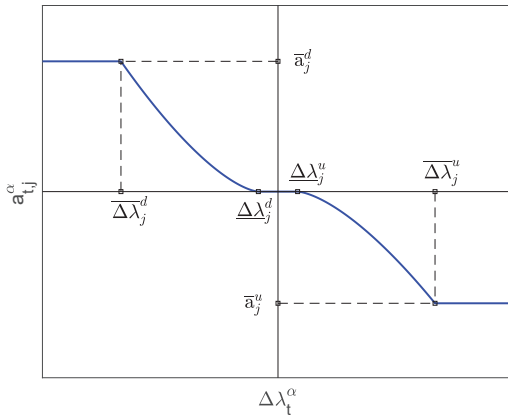


Fig. 1. Willingness parameter $\mathbf{a}_{t,j}^\alpha$ for time-varying electricity price $\Delta\lambda_t^\alpha$. Positive prices lead to up-regulation (i.e., $\alpha \equiv u$), while negative prices induce down-regulation (i.e., $\alpha \equiv d$). The parameters $\underline{\Delta\lambda}_j^\alpha$ and $\bar{\Delta\lambda}_j^\alpha$ determine the dead-band and the saturation prices for each end-users' category j .

$$\mathbf{a}_{t,j}^\alpha = 0 \quad |\Delta\lambda_t^\alpha| < \underline{\Delta\lambda}_j^\alpha \quad (2)$$

When the delta price increases beyond the dead-band, the pool of consumers starts reacting, which is modelled as

follows:

$$\mathbf{a}_{t,j}^\alpha = \bar{\mathbf{a}}_j^\alpha \left(\frac{\Delta\lambda_t^\alpha - \underline{\Delta\lambda}_j^\alpha}{\bar{\Delta\lambda}_j^\alpha - \underline{\Delta\lambda}_j^\alpha} \right)^\gamma \quad \underline{\Delta\lambda}_j^\alpha \leq |\Delta\lambda_t^\alpha| \leq \bar{\Delta\lambda}_j^\alpha \quad (3)$$

Furthermore, we assume that, beyond a certain price, i.e., $\bar{\Delta\lambda}_j^\alpha$, the pool cannot provide additional flexibility because of the rebound effect and the un-curtailable load, as discussed in [8]. Therefore, $\mathbf{a}_{t,j}^\alpha$ becomes constant:

$$\mathbf{a}_{t,j}^\alpha = \bar{\mathbf{a}}_j^\alpha \quad |\Delta\lambda_t^\alpha| \geq \bar{\Delta\lambda}_j^\alpha \quad (4)$$

To account for the stochasticity and the diversity among consumers even from the same category of end-users, the six parameters defining the dead-band and saturation, shown in Fig. 1, are treated as normally-distributed random variables. In Subsection V-A, the statistical properties and a simulation framework will be introduced to generate a pool of consumers for each end-users' category.

III. UP- AND DOWN-FLEXIBILITY: DETERMINISTIC CASE

The ultimate goal of this study is to estimate the amount of demand flexibility that can be provided by different categories of end-users, under a time-varying pricing scheme in the presence of stochasticity in consumers' willingness. By having the stochastic model of the consumers' reaction to the price signal and the assumptions made in the previous section, it is possible to formulate an optimisation problem for the REUs to estimate their flexibility. The formulation is developed based on the conservative assumption that a perfect rebound exists due to practical reasons and end-users' comfort. In fact, more than 90% of the flexibility resources at the residential premises is provided by appliances with shiftable load (e.g., heating, ventilation, and air conditioning systems, clothes dryers, and so on) [38]. Therefore, the rebound effect will be an inevitable aspect of demand flexibility modelling, although it adversely affects the overall flexibility. As consumers might not be willing to increase their overall daily consumption, which might result in higher electricity bills, a perfect load shifting is preferred in the model that must be completed within a certain time period. While this condition might further decrease the overall flexibility of the load demand, it provides a more realistic model of consumers' behaviour, which consequently improves the accuracy of the estimated flexibility. Since the TSO does not have direct access to the individual loads, and consumers only react to the delta prices submitted by the TSO, flexibility should be estimated from consumers' perspective. Therefore, the model is formulated as a minimisation of the daily cost of electricity consumption for each end-users' category, as shown below:

$$\min_{L_{t,j}^\alpha} \sum_{t=1}^T (\lambda^{\text{base}} + \Delta\lambda_t^u + \Delta\lambda_t^d) \sum_{j=1}^J (L_{t,j}^{\text{base}} + L_{t,j}^d - L_{t,j}^u) \quad (5a)$$

$$\text{s.t.} \quad -\mathbf{r}_j^\alpha \leq L_{t+1,j}^\alpha - L_{t,j}^\alpha \leq \mathbf{r}_j^\alpha \quad \forall t, j, \alpha \quad (5b)$$

$$0 \leq L_{t,j}^d \leq u_{t,j}^d (\mathbf{L}_{t,j}^{\text{max}} - \mathbf{L}_{t,j}^{\text{base}}) \mathbf{a}_{t,j}^d \quad \forall t, j \quad (5c)$$

$$0 \leq L_{t,j}^u \leq u_{t,j}^u (\mathbf{L}_{t,j}^{\text{base}} - \mathbf{L}_{t,j}^{\text{min}}) \mathbf{a}_{t,j}^u \quad \forall t, j \quad (5d)$$

$$\sum_{t'=(t-1)R_j+1}^{(t-1)R_j+R_j} (L_{t',j}^d - L_{t',j}^u) = 0 \quad (5e)$$

$$\forall t : [t \in T, (tR_j \leq \tau)], j$$

$$u_{t,j}^d + u_{t,j}^u \leq 1 \quad \forall t, j \quad (5f)$$

$$y_{t,j}^\alpha - z_{t,j}^\alpha = u_{t,j}^\alpha - u_{t-1,j}^\alpha \quad \forall t, j, \alpha \quad (5g)$$

$$y_{t,j}^\alpha + z_{t,j}^\alpha \leq 1 \quad \forall t, j, \alpha \quad (5h)$$

$$\sum_{t=1}^{\tau} y_{t,j}^\alpha \leq \mathbf{n}_j^\alpha \quad \forall j, \alpha \quad (5i)$$

$$\sum_{t'=t}^{t+\underline{\mathbf{d}}_j^\alpha} u_{t',j}^\alpha \geq \underline{\mathbf{d}}_j^\alpha y_{t,j}^\alpha \quad (5j)$$

$$\forall t : [t \in T, (t + \underline{\mathbf{d}}_j^\alpha < \tau)], j, \alpha$$

$$\sum_{t'=t}^{t+\overline{\mathbf{d}}_j^\alpha} z_{t',j}^\alpha \geq y_{t,j}^\alpha \quad (5k)$$

$$\forall t : [t \in T, (t + \overline{\mathbf{d}}_j^\alpha < \tau)], j, \alpha$$

The objective function in Eq. (5a) calculates the cost of each end-users' category for purchasing electricity within the time period τ (i.e., $\tau = 24$ hours). The constraints are formulated as follows: Eq. (5b) is related to the up- and down-ramp limits of the flexible loads, which are represented for each end-users' category j by the ramp-rate parameter \mathbf{r}_j^α ; Eq. (5c) and (5d) impose lower and upper bounds on the amount of flexibility that can be provided by each end-users' category. Note that the minimum and maximum load for each category j at time t , i.e., $\mathbf{L}_{t,j}^{\min}$ and $\mathbf{L}_{t,j}^{\max}$, represent the lowest and highest possible consumption that each end-users' category can sustain at time t . In other words, they define the demand flexibility that can be achieved from each end-users' category in a specific time. Eq. (5e) implements the energy conservation rule for each end-users' category, as explained at the beginning of this section. In this constraint, the parameter R_j consists of the maximum rebound delay by which the load shifting must be completed for each end-users' category j . Eq. (5f) ensures that only one type of flexibility (i.e., up- or down-regulation) is provided by a specific end-users' category j at time t ; Eq. (5g) and (5h) represent the flexibility activation and deactivation for each end-users' category j at time t ; Eq. (5i) enforces a limit on the number of times that a certain end-users' category can be activated in a day. In Eq. (5i), it is assumed that only a certain number of processes can be shifted within the day; Eq. (5j) and Eq. (5k) refer to the minimum and maximum duration for which the load response can be sustained. Obviously, many of the parameters depend on the end-users' category, and hence the above optimisation will be solved for a certain number of consumers in each end-users' category, representing the characterisations and the statistical variability in that end-users' category.

IV. UP- AND DOWN-FLEXIBILITY: CHANCE-CONSTRAINED PROGRAMMING

Due to the importance of AS in the power system operation and the stochastic nature of the REUs, it is valuable for the TSO to quantify the risk in demand flexibility and include

it in the decision-making process. To do so, the deterministic optimisation formulation from the previous section is converted to a chance-constrained (CC) programming. This way, it is plausible to deal with the level of risk associated with the provision of a certain amount of demand flexibility. The CC formulation ensures that the probability of meeting a certain constraint is above a preferred confidence level [39] by restricting the feasible solution space. The CC programming has been used in the past to solve different power system problems. For instance, it has been applied to optimal storage sizing in [40], and to generate optimal price signals for DR programs from the householders in [41]. Also, in [42], such a method has been used in an optimal power flow model of a 30-bus system to schedule generation and reserve, where controllable loads have been considered as thermal energy storage units.

From our model formulation, it can be seen that each end-users' category acts independently to minimise its operation cost. In Eq. (3), $\mathbf{a}_{t,j}^\alpha$ is defined as a function of the electricity price, consumers' preferences, end-users' category, and time of the day. Even though this parameter does not explicitly depend on its previous values in time, the load price-response is made time-dependent by way of constraints (5e)-(5k), which directly limit the provision of flexibility from consumers over time. For instance, Eq. (5k) prevents the loads from providing flexibility beyond a certain period of time, in particular, $\overline{\mathbf{d}}_j^\alpha$ hours. This way, the provision of flexibility by loads at one hour depends of its previous values. Time dependency is also enforced by limiting the maximum number of load flexibility activations or by modelling the rebound effect, as explained in Section III. On the other hand, as $\mathbf{a}_{t,j}^\alpha$ does not depend on its previous values in time, it is possible to evaluate each constraint independently by using a disjoint CC method. From the formulation of the deterministic model, the flexibility was limited by:

$$\begin{aligned} L_{t,j}^d &\leq u_{t,j}^d (\mathbf{L}_{t,j}^{\max} - \mathbf{L}_{t,j}^{\text{base}}) \mathbf{a}_{t,j}^d \quad \forall t, j \\ L_{t,j}^u &\leq u_{t,j}^u (\mathbf{L}_{t,j}^{\text{base}} - \mathbf{L}_{t,j}^{\min}) \mathbf{a}_{t,j}^u \quad \forall t, j \end{aligned} \quad (6)$$

In order to apply CC programming, $\mathbf{a}_{t,j}^\alpha$ is treated as a random variable and denoted by $\tilde{\mathbf{a}}_{t,j}^\alpha$. It is a function of input parameters $\underline{\Delta\lambda}_j^\alpha$, $\overline{\Delta\lambda}_j^\alpha$, and $\overline{\mathbf{a}}_j^\alpha$, as given in Eq. (3). As argued in [43], [44], the input parameters are assumed to be normally distributed because of their dependence on a large number of individual human behaviour:

$$\begin{aligned} L_{t,j}^d &\leq u_{t,j}^d (\mathbf{L}_{t,j}^{\max} - \mathbf{L}_{t,j}^{\text{base}}) \tilde{\mathbf{a}}_{t,j}^d \quad \forall t, j \\ L_{t,j}^u &\leq u_{t,j}^u (\mathbf{L}_{t,j}^{\text{base}} - \mathbf{L}_{t,j}^{\min}) \tilde{\mathbf{a}}_{t,j}^u \quad \forall t, j \end{aligned} \quad (7)$$

The right-hand side of Eq. (7) can be re-written in a compact form, as follows:

$$\mathcal{A}_{t,j}^d \equiv u_{t,j}^d (\mathbf{L}_{t,j}^{\max} - \mathbf{L}_{t,j}^{\text{base}}) \tilde{\mathbf{a}}_{t,j}^d \quad (8a)$$

$$\mathcal{A}_{t,j}^u \equiv u_{t,j}^u (\mathbf{L}_{t,j}^{\text{base}} - \mathbf{L}_{t,j}^{\min}) \tilde{\mathbf{a}}_{t,j}^u \quad (8b)$$

$$L_{t,j}^\alpha \leq \mathcal{A}_{t,j}^\alpha \quad \forall t, j \quad (8c)$$

In CC programming, each constraint needs to be satisfied for a probability higher than a predefined theoretical confidence level β_{th} , where th means that it is the theoretical value imposed in the formulation.

$$Pr\left(L_{t,j}^{\alpha} \leq \mathcal{A}_{t,j}^{\alpha}\right) \geq \beta_{th} \quad (9)$$

Adding mean $\mu(\cdot)$ and standard deviation $\sigma(\cdot)$ of $\mathcal{A}_{t,j}^{\alpha}$ to the formulation, we will have:

$$Pr\left(\frac{L_{t,j}^{\alpha} - \mu_{\mathcal{A}_{t,j}^{\alpha}}}{\sigma_{\mathcal{A}_{t,j}^{\alpha}}} \leq \frac{\mathcal{A}_{t,j}^{\alpha} - \mu_{\mathcal{A}_{t,j}^{\alpha}}}{\sigma_{\mathcal{A}_{t,j}^{\alpha}}}\right) \geq \beta_{th} \quad (10)$$

If $\mathbf{a}_{t,j}^{\alpha}$ follows a normal distribution, then it is possible to define the standard score as z_{α} :

$$Pr\left(\frac{L_{t,j}^{\alpha} - \mu_{\mathcal{A}_{t,j}^{\alpha}}}{\sigma_{\mathcal{A}_{t,j}^{\alpha}}} \leq z_{\alpha}\right) \geq \beta_{th} \quad (11a)$$

$$1 - Pr\left(\frac{L_{t,j}^{\alpha} - \mu_{\mathcal{A}_{t,j}^{\alpha}}}{\sigma_{\mathcal{A}_{t,j}^{\alpha}}} \geq z_{\alpha}\right) \geq \beta_{th} \quad (11b)$$

where the cumulative distribution function (CDF), called Φ , can be estimated as follows:

$$1 - \Phi\left(\frac{L_{t,j}^{\alpha} - \mu_{\mathcal{A}_{t,j}^{\alpha}}}{\sigma_{\mathcal{A}_{t,j}^{\alpha}}}\right) \geq \beta_{th} \quad (12)$$

Eq. (12) can be further rearranged:

$$L_{t,j}^{\alpha} \leq \mu_{\mathcal{A}_{t,j}^{\alpha}} + \sigma_{\mathcal{A}_{t,j}^{\alpha}} \Phi^{-1}(1 - \beta_{th}) \quad (13)$$

By defining $\Phi^{-1}(1 - \beta_{th})$ as $\Phi_{\beta_{th}}^{-1}$, the constraints can be written as:

$$L_{t,j}^d \leq \mu_a^d u_{t,j}^d (\mathbf{L}_{t,j}^{\max} - \mathbf{L}_{t,j}^{\text{base}}) + \sigma_a^d u_{t,j}^d (\mathbf{L}_{t,j}^{\max} - \mathbf{L}_{t,j}^{\text{base}}) \Phi_{\beta_{th}}^{-1} \quad (14a)$$

$$L_{t,j}^u \leq \mu_a^u u_{t,j}^u (\mathbf{L}_{t,j}^{\text{base}} - \mathbf{L}_{t,j}^{\min}) + \sigma_a^u u_{t,j}^u (\mathbf{L}_{t,j}^{\text{base}} - \mathbf{L}_{t,j}^{\min}) \Phi_{\beta_{th}}^{-1} \quad (14b)$$

According to the value of the binary variable $u_{t,j}^{\alpha}$, two scenarios can be identified:

- **Scenario I:** $u_{t,j}^{\alpha} = 0$, where:

$$L_{t,j}^{\alpha} = 0 \quad (15)$$

In this scenario, the flexibility is zero.

- **Scenario II:** $u_{t,j}^{\alpha} = 1$, where:

$$L_{t,j}^d \leq \mu_a^d (\mathbf{L}_{t,j}^{\max} - \mathbf{L}_{t,j}^{\text{base}}) + \sigma_a^d (\mathbf{L}_{t,j}^{\max} - \mathbf{L}_{t,j}^{\text{base}}) \Phi_{\beta_{th}}^{-1} \quad (16a)$$

$$L_{t,j}^u \leq \mu_a^u (\mathbf{L}_{t,j}^{\text{base}} - \mathbf{L}_{t,j}^{\min}) + \sigma_a^u (\mathbf{L}_{t,j}^{\text{base}} - \mathbf{L}_{t,j}^{\min}) \Phi_{\beta_{th}}^{-1} \quad (16b)$$

According to Eq. (16a) and (16b), the amount of flexibility is bounded by a certain value that takes into account the mean and standard deviation of $\mathbf{a}_{t,j}^{\alpha}$ and the quantile of a standard normal variable. The latter will depend on the predefined theoretical confidence level, i.e., β_{th} , and the estimated flexibility by this method will be guaranteed at that confidence level. Therefore, it will help TSO to make an informed decision considering its risk.

V. SIMULATION STUDY AND DISCUSSION

To show the effectiveness of the proposed model, a simulation study is carried out using actual data, which are provided by Elforbrugspanel in 2008. Data is collected by Energinet (the Danish transmission system operator) and Dansk Energi (the Danish advocacy group for energy companies) by monitoring hourly electricity demand for a selected pool of consumers in every Danish municipality [45]. The selected pool has been defined to represent the national demand. 2106 meters have been installed to study the residential, agricultural, industrial and commercial electricity demand in this project. The aggregated data of each end-users' category has been reported monthly to Elforbrugspanel. The main output of the project has been the calculation of the average of the hourly individual electricity demand for 29 end-users' category.

The proposed formulation can possibly work with different AS markets and time-frames in the order of minutes to hours, as long as the required data with the right time resolution is available. In our simulation studies, we consider balancing services that are procured one day in advance and use data for the hourly average consumption for 29 end-users' categories, a list of which is given in Table I. This way, the estimated delta prices are submitted to the REUs' EMS in a single shot 24 hours ahead, and the problem is solved once for all types of loads. In order to compound the aggregated behaviour of the consumers, the actual consumption of each category is weighted by the total number of consumers in that category, which is obtained from [46]. The data used in the simulations is also available in [47]. The simulation starts by generating a pool of consumers of diverse flexibility in subsection V-A. Then, the normality assumption of the consumers' willingness, $\mathbf{a}_{t,j}^{\alpha}$, is checked for the CC optimisation problem. In subsection V-D, the deterministic and CC optimisations are solved for different load categories with two different confidence levels. In subsection V-D, the impact of the confidence level on the results is analysed and the results of CC optimisation are validated in subsection V-E. Finally, in subsection V-F, the impact of different rebound effects on the results is investigated.

A. Generating a pool of consumers' willingness

In the first part of the simulation, a pool of consumers is created with different preferences, i.e., $\mathbf{a}_{t,j}^{\alpha}$, followed by checking the normality of their behaviour, as shown in the flowchart of Fig. 2. Then, the CC optimisation problem is solved with the given theoretical confidence level, β_{th} , to quantify the aggregated load flexibility.

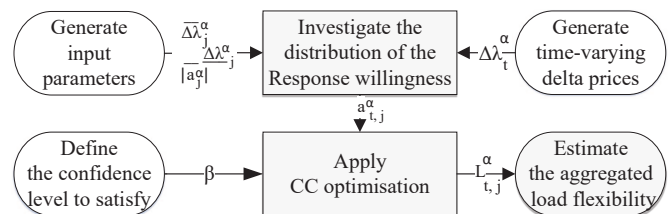


Fig. 2. Conceptual flowchart of the simulation study.

Prior to that, however, delta prices should be generated. As mentioned in Section II, a certain delta price set will be communicated from the system operator to the REUs to create a change in their consumption. In Eq. (5a), the baseline electricity price, λ^{base} , is set to 225 DKK cent/kWh [48], the hourly delta price set is randomly generated by following a uniform distribution. The magnitude of delta prices (i.e., absolute value) is within the range of $[20, 75]$ cent DKK/kWh, following the rule defined in Eq. (1) and Eq. (3). As one can see, the delta price range is set to be well beyond the dead-band and below the flexibility saturation in consumers' willingness to avoid violating the upper and lower limits [8]. In fact, it is counterproductive for the TSO to submit an insignificant price (i.e., lower than the dead-band price) to the pool of consumers, as no reaction will be achieved. On the other hand, it is economically inconvenient for the TSO and consumers to submit an excessive price (i.e., higher than the saturation price), as the same price response can be achieved with a smaller price. Considering the limited accuracy of the estimated prices due to the unpredictable nature of AS requirements, the delta prices is unknown to a large extent. Therefore, it is reasonable to treat it like a normally-distributed random parameter. In this study, we simulate the aggregated flexibility response for 5000 different daily profiles of delta prices. This is to estimate the range of potential flexibility in each hour of the day, accounting for the stochasticity in delta prices and quantifying the risk for the system operator in exploiting load demand flexibility. In the future, these delta prices might be generated by another optimisation problem [23].

In the simulations, we will refer to the absolute value of the maximum willingness parameter $|\bar{a}_{t,j}^{\alpha}|$. This is because we consider that the magnitude (i.e., absolute value) of the maximum willingness parameter will be the same to provide up- or down-regulation for each end-users' category j . For this reason, we just provide the absolute value of $\bar{a}_{t,j}^{\alpha}$ to calculate $\mathbf{a}_{t,j}^{\alpha}$. However, $\mathbf{a}_{t,j}^u$ is supposed to be negative and $\mathbf{a}_{t,j}^d$ is positive, as shown in Fig.1. Therefore, we calculate $\mathbf{a}_{t,j}^u$ and $\mathbf{a}_{t,j}^d$ from Eq. (3), using the same magnitude of $|\bar{a}_{t,j}^{\alpha}|$ but with opposite signs. The mean and standard deviation of the input parameters used in the simulation study, i.e., $\Delta\lambda_j^{\alpha}$, $\Delta\lambda_j^{\alpha}$, $|\bar{a}_j^{\alpha}|$, are reported in Table I. The parameters are used to generate random numbers using a normal distribution. Due to data scarcity for different end-users' categories, input parameters are assumed to be the same for all time instances and up- and down-regulation. In order to determine the values of $|\bar{a}_{t,j}^{\alpha}|$ for each end-users' category, reference [49] is used, where the amount of maximum flexibility is quantified for several consumers' sectors in Denmark. Such estimates are compared to the consumption that we previously calculated for each end-users' category from the data set. For instance, according to [49], cement manufacturing and iron foundries are able to provide 16 MW load reduction. From the consumption we previously calculated from the data set, the total electricity consumption of these consumers' categories is 30 MW. Therefore, a maximum willingness parameter of 0.5 (i.e., 50% of the total consumption) is estimated for these

categories (i.e., basic metal and construction). In different studies, e.g., [58], [59], various sectors and countries have been investigated in price elasticity, whose concept is discussed in [60]. To include diversity in the willingness parameters for those sectors whose estimate in [49] is provided only for aggregated loads, this concept of price elasticity is used. Because of the lack of information, the willingness parameters of the remaining sectors are randomly chosen. Moreover, $\sigma_{\bar{a}_j^{\alpha}}$ is defined in a way that the values of $|\bar{a}_j^{\alpha}|$ are maintained between 0 and 1. When the value generated by the distribution function exceeds the higher or lower limits (1 for higher and 0 for lower limits), another random number is re-drawn from the normal distribution. The choice of $\Delta\lambda_j^{\alpha}$ is approximately selected based on the nature of different end-users' categories [24]. For instance, we assumed that industries might behave such that they prevail the continuity of their service, unless very high delta prices are offered. Similarly, $\Delta\lambda_j^{\alpha}$ values are intuitively determined. For the case of $\Delta\lambda_j^{\alpha}$, the value of σ is chosen by considering a normal distribution of prices and that each price has to be bigger than zero. 5000 samples of $|\bar{a}_j^{\alpha}|$ are generated for each category of end-users and time. The results of a sample end-users' category are shown in Fig. 3. The number of samples is chosen in a way to statistically represent the variability of consumers' willingness in every end-users' category. These values are later used in the optimisation studies, both deterministic and CC problems, to estimate the aggregated load flexibility.

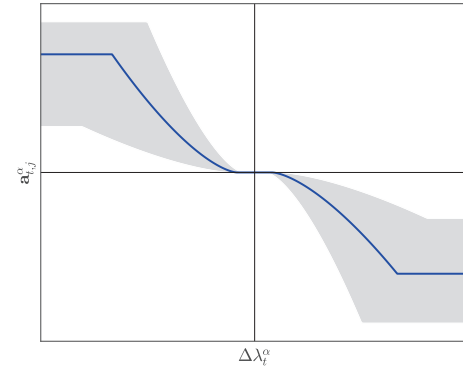


Fig. 3. Range of $\mathbf{a}_{t,j}^{\alpha}$ achieved for a sample end-users' category for different $\Delta\lambda_t^{\alpha}$.

In the next step, we investigate the normality of $\mathbf{a}_{t,j}^{\alpha}$ in order to justify the application of CC programming. Eq. (3) is defined as the ratio of two normal components, namely $(\Delta\lambda_t^{\alpha} - \Delta\lambda_j^{\alpha})$ and $(\Delta\lambda_j^{\alpha} - \Delta\lambda_j^{\alpha})$, which might lead to a non-normal distribution. In Fig. 4, a statistical analysis using QQ plot and histograms of \mathbf{a}_j^{α} is carried out for a sample load category j and up- and down-flexibility at a specific time. In the QQ plots, the two vertical lines represent ± 2 standard deviations of the data, meaning that the values within those lines are 95% of the data. Fig. 4 shows that the behaviour of up- and down-willingness is approximately normal due to the dominating variance of \bar{a}_j^{α} in Eq. (3).

TABLE I
AVERAGE VALUES $\mu(\cdot)$ AND STANDARD DEVIATIONS $\sigma(\cdot)$ OF $\Delta\lambda_j$, $\Delta\lambda_j$ [DKK CENT/KWH] AND $|\bar{a}_j^\alpha|$ [P.U.]; RAMP [kW], FLEXIBILITY ACTIVATIONS [P.U.], FLEXIBILITY DURATION [H] AND REBOUND DELAY [H] PARAMETERS FOR DIFFERENT END-USERS' CATEGORIES.

End-users' category	$ \bar{a}_j^\alpha $		Reference	$ \Delta\lambda_{t,j}^\alpha $		$ \Delta\lambda_{t,j}^\alpha $		\bar{r}_j^α	n_j^α	\underline{d}_j^α	\bar{d}_j^α	R_j	Reference
	μ	σ		μ	σ	μ	σ						
Apartment without heating	0.4	0.10	[45], [49], [50]	5	1.1	105	11	3	7	1	6	8	[51]
House without heating	0.5	0.11	[45], [49], [50]	6	1.2	100	12	3	8	1	6	8	[51]
House with heating	0.6	0.10	[45], [49], [50]	6	1.0	105	15	5	20	1	2	3	[51]
Cottage	0.5	0.13	—	6	1.1	100	13	4	13	1	4	6	—
Gardening	0.6	0.10	—	10	1.0	110	10	3	17	1	3	6	—
Agriculture without heating	0.5	0.12	—	14	1.3	110	10	4	6	1	4	8	[52]
Agriculture with heating	0.7	0.08	—	14	1.3	110	11	5	20	1	2	3	[52]
Food	0.4	0.10	[45], [49], [53]	12	1.8	130	14	2	12	1	2	3	[51]
Basic metal	0.5	0.12	[45], [49], [53]	11	1.5	120	14	3	4	1	5	12	[51]
Wood	0.3	0.05	[45], [49], [53]	14	1.7	120	12	3	4	1	4	12	[51]
Textile	0.3	0.06	[45], [49], [53]	13	1.2	120	10	3	5	1	6	12	—
Paper	0.4	0.10	[45], [49], [53]	15	1.0	120	15	3	3	1	4	12	[51], [54]
Non-metallic	0.3	0.06	[45], [49]	12	1.1	120	10	3	5	1	8	12	—
Chemical	0.5	0.13	[45], [49], [53]	16	1.0	130	11	2	2	1	3	8	[51], [55]
Other industries	0.3	0.05	[45], [49], [53]	11	1.0	120	10	3	5	1	8	12	—
Construction	0.5	0.10	—	8	1.2	120	14	4	2	1	3	8	[51]
Retail	0.4	0.10	[45], [49]	8	1.2	120	14	6	4	1	3	4	[56]
Wholesale	0.4	0.10	[45], [49]	8	1.2	120	14	5	9	1	3	4	[56]
Bank	0.3	0.05	[45], [49]	17	2.0	150	10	6	8	1	2	3	—
Utility	0.3	0.06	—	14	1.4	110	10	3	3	1	2	3	[51]
Sewerage	0.3	0.05	—	17	2.0	150	12	3	3	1	2	3	[51]
Cultural	0.5	0.13	[45], [49]	10	1.2	140	10	6	11	1	3	4	—
Restaurant	0.6	0.10	[45], [49]	7	1.0	110	10	5	8	1	3	4	[51]
Health	0.3	0.06	—	17	2.0	150	11	3	4	1	4	6	—
Education	0.5	0.12	[45], [49]	10	1.2	140	16	5	20	1	2	3	—
Social	0.5	0.13	[45], [49]	10	1.2	140	15	4	12	1	3	4	—
Postal	0.6	0.10	[45], [49]	10	1.2	140	12	5	15	1	3	4	—
Public	0.6	0.11	[45], [49]	10	1.2	140	13	6	13	1	2	3	—
Public light	0.3	0.02	—	17	2.0	150	12	5	3	1	2	3	[57]

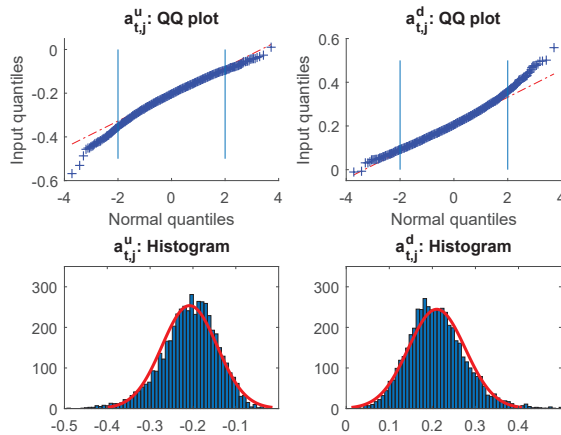


Fig. 4. QQ plots and histograms for $a_{t,j}^u$ and $a_{t,j}^d$ of a sample end-users' category j at a specific time t .

B. Selection of γ

In [35], the price responsiveness of consumers is modelled as a linear function, which is equivalent to a value of γ equal to 1 in Eq. (3). In spite of that, it is reasonable to assume that consumers might be more inclined to alter their consumption profile when they receive big delta prices, as also suggested in [36]. The value of γ will be limited by the fact that consumers have different sensitivities to prices and some of them might always be responsive to achieve cost minimisation. In Fig. 5, the distribution of $a_{t,j}^\alpha$ is analysed for different values of γ

(i.e., 1, 1.5 and 2). It is clear from the figure that a reasonable choice of γ does not compromise the normality assumption. In this paper, γ is equal to 1.5.

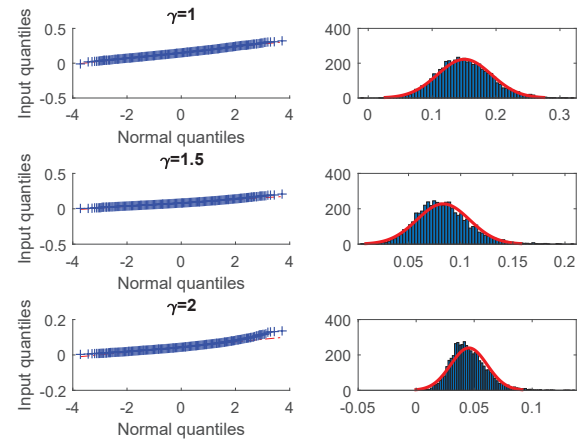


Fig. 5. QQ plot and histogram for generic $a_{t,j}^\alpha$ of a sample end-users' category j at a specific time t for values of γ equal to 1, 1.5 and 2.

C. Explanation of the consumers' constraints parameters

In the simulations, $L_{t,j}^{\min}$ and $L_{t,j}^{\max}$ are calculated from the available data set [45], by identifying the minimum and maximum values of the historical electricity consumption for each time t and end-users' category j . This method is preferred in this study as it is the only information that was available at the time. Following a similar approach, $L_{t,j}^{\text{base}}$ is calculated

from the data set by averaging the consumption of each end-users' category at time t . Parameters related to the consumers' constraints (e.g., ramp, flexibility provision duration and flexibility activation times) are estimated due to the current lack of more detailed information and provided in Table I. The ramp parameter r_j^α is determined from the consumption data set, as $r_j^\alpha = \tilde{r}_j^\alpha \max_{1 \leq t \leq \tau} (L_{t,j}^{\max})$, where \tilde{r}_j^α is a parameter that depends on the type and characteristics of the loads of each end-users' category j . Considering hourly resolution of data and proposed formulation, it is reasonable to assume that \tilde{r}_j^α will not be very restrictive since loads can change relatively fast. In fact, the majority of the loads have faster dynamics than an hour, i.e., they can go from 0 to 100% consumption in less than an hour. For the consumers' categories with mainly thermal loads (e.g., public [49]) and whose processes can be shifted in time (e.g., paper [54]), a larger \tilde{r}_j^α is assumed. For the industrial consumers, however, it will be more restrictive. In order to determine the amount of activation times for each end-users' category n_j^α , it is assumed that the industrial consumers have generally less shiftable processes compared to the residential and commercial consumers. Therefore, n_j^α for industrial consumers is considered smaller than for residential consumers. By generally accounting on the flexibility from ventilation, heating and air conditioning (HVAC), it is feasible for the residential and commercial consumers to be activated and deactivated several times during the day without technical constraints. On the other hand, a waste water treatment facility from the industrial sector might be the only shiftable process, limiting the overall consumption flexibility. In determining d_j^α , it is assumed that end-users' category can provide flexibility for a minimum duration of 1 hour, as HVAC is present in almost every end-users' category. Regarding the choice of the maximum flexibility duration values in Table I, the commercial consumers are assumed to be mainly affected by the thermal dynamics of HVAC [49]. For the industrial and residential consumers, longer dynamics are expected, as their loads are not only thermal and they might have different characteristics (e.g., electric vehicle charging, laundry machine and so on). In the simulation studies, the case of perfect daily rebound is solved for each end-users' category (i.e., $R_j = 23$ for each j). Afterwards, a conservative case is considered by applying strict rebound effects, given in Table I, in order to evaluate the impact of the rebound on the overall flexibility. To determine the parameter R_j for the case of strict rebound, it is assumed that the end-users' flexibility is mainly constrained by the thermal dynamics of their loads. However, there are cases like the paper industry where production processes can be shifted to other times of the day [54]. For end-users' categories where processes can be shifted within the day, the rebound constraint is relaxed. Also, for the agricultural consumers, R_j is estimated by accounting for the processes involving animal waste treatment, irrigation and curing tobacco [52].

D. Up- and down-flexibility estimation

In this section, the CC optimisation problem is solved for different theoretical confidence levels using the $a_{t,j}^\alpha$ values from Section V-A.

• Low-risk case

For a conservative simulation study, $\beta_{th} = 0.95$ is selected as theoretical confidence level. It implies that, globally, the constraints in Eq. (5c) and (5d) will be respected with a probability that is equal or higher than 95%. In other words, it guarantees that the estimated flexibility from the consumers, given their stochastic behaviour, will be achieved 95% of the time or higher.

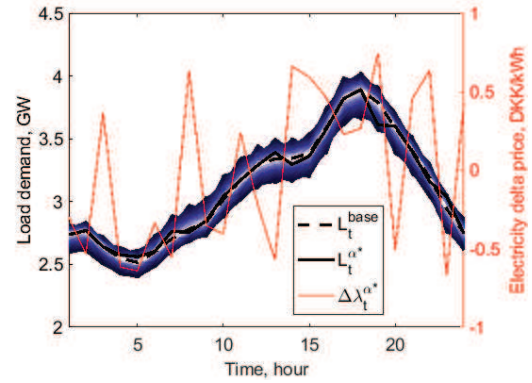


Fig. 6. Flexibility achieved for different delta prices by CC optimisation for $\beta_{th} = 0.95$ considering daily rebound: baseline consumption, flexibility for the reference delta price $\Delta\lambda_t^{\alpha*}$, and the delta price.

In Fig. 6, the achievable flexibility for different prices is shown in relation to the baseline consumption for $\beta_{th} = 0.95$. It emerges that the maximum flexibility is about 7% of the hourly load demand. It is also noticeable that the flexibility in the early morning is mainly for up-regulation, while the down-regulation potential seems to be small, i.e., around 3% of the hourly load demand. Although such a result may appear counter-intuitive, it is due to the selected values of $L_{t,j}^{\min}$ and $L_{t,j}^{\max}$ that are used in the simulation studies. They are extracted from annual data by finding the minimum and maximum consumption values of each end-users' category at each hour of the day. Since the data set at hand does not include the impact of consumers' response to the prices, the maximum load in early hours is very close to the average consumption, which resulted in lower down-flexibility in the simulation results. In the future, advanced methods can be developed to calculate these parameters by collecting aggregated data from REUs in response to the delta prices. The correlation between delta prices and flexibility is -0.73 , confirming a strong negative correlation between the two parameters. The correlation does not reach -1 because of the constraints applied to the minimisation problem and the different amount of flexibility available for up- and down-regulation. In order to verify the correlation between flexibility and delta prices visually, the flexibility obtained in response to a randomly-selected daily delta prices, i.e., $\Delta\lambda_t^{\alpha*}$, is shown in Fig 6. It can be noticed that the highest amount of down-regulation (i.e., increased consumption) is achieved at hour 23:00, with 3.6% increase in demand, corresponding to the biggest negative delta price. The highest amount of up-regulation is achieved at hour 19:00 with 5.8% decrease in total demand, coinciding with a relatively large positive delta price.

• High-risk case

The CC optimisation for 5000 delta prices is repeated for $\beta_{th} = 0.50$. As expected, the up- and down-flexibility patterns are identical to the “low-risk case.” However, their magnitude increases substantially for all hours, as shown in Fig. 7. It can be seen that the flexibility range raises by 76% compared to the “low-risk case.” At hour 23:00, the demand is expected to increase by about 7% in response to the given price, while 12.2% decrease in demand is observed at hour 19:00. The simulation results show that the TSO might over-estimate the flexibility potential if the associated risk is not considered in the formulation. It will, in turn, result in unsuccessful demand flexibility procurement in the real-time operation.

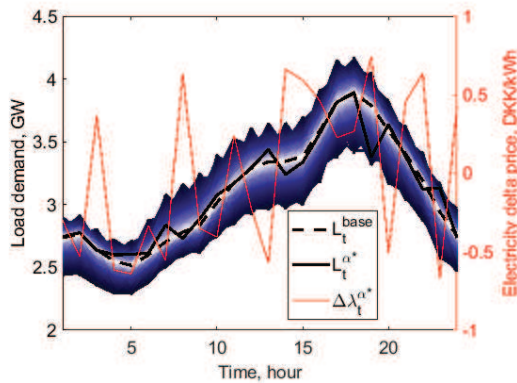


Fig. 7. Flexibility achieved for different delta prices by CC optimisation for $\beta_{th} = 0.50$ considering daily rebound: baseline consumption, flexibility for the reference delta price $\Delta\lambda_t^{\alpha*}$, and the delta price.

E. Validation of CC formulation

In this sub-section, we investigate the quality of the CC solutions for the case of daily rebound. As it was mentioned earlier, the CC solutions are valid only if the actual confidence level (i.e., β_{ac} achieved in the Monte Carlo simulation study) is bigger than or equal to the theoretical confidence level (i.e., β_{th} imposed on the formulation and associated simulation study) of the CC programming. To do so, we need to impose a theoretical confidence level and solve the CC formulation for a given price set. From the results, the flexibility $L_{t,j}^{\alpha}$ is obtained. Afterwards, this value is used in Eq. (5c) and (5d) to investigate how many times the constraints are violated. In Eq. (5c) and (5d), $\mathbf{a}_{t,j}^{\alpha}$ is the generated pool of consumers' willingness discussed in subsection V-A. Since we are dealing with thousands of constraints in this simulation, while our intent is to provide a readable plot of the results, we calculate the mean value of the actual confidence level of the various constraints. This process is repeated for different values of theoretical confidence level, i.e., $\beta_{th} \in [0.1, 0.98]$, and the mean values of β_{ac} are plotted in Fig. 8(a) in comparison to the β_{th} imposed. From the figure, it can be seen that the actual confidence level is always higher than the theoretical counterpart. Therefore, it can be concluded that the constraints are always satisfied for the given confidence level, and that the normality assumption of $\mathbf{a}_{t,j}^{\alpha}$ was correct. Moreover, Fig. 8(a) shows that the CC programming behaves more conservatively on the lower range of β_{th} , where the actual confidence level is always greater than the theoretical one (e.g., for $\beta_{th} = 0.50$, the

actual confidence level is 0.54). This is because the constraints are loosely confined for small β_{th} values, which result in more availability of load demand to provide flexibility.

Also, in order to understand the value of using CC, we include a study where we compare the different performances of the deterministic and stochastic cases. Therefore, we solve the stochastic and deterministic formulations, where in the latter it is imposed a β_{th} of 0.95. Afterwards, we calculate for each formulation what is the percentage of the constraints that achieve a certain β_{ac} . In Fig. 8(b), the results are provided through probability density functions. For the actual β_{ac} show that, for the stochastic case (i.e., blue line), 92% of the constraints have a β_{ac} that is slightly above 0.95. In a few instances, $\beta_{ac} = 1$ is obtained because of the condition imposed in Eq. (5e) and the rebound effect. For the deterministic case (i.e., black line), however, the actual confidence level lies below 92% for 95% of the constraints and is lower than the one obtained by the CC formulation. These results prove that quantifying the risk and trying to maintain a specific level of certainty is of paramount importance for the TSO in real-time operation, which is provided by the CC formulation in this study.

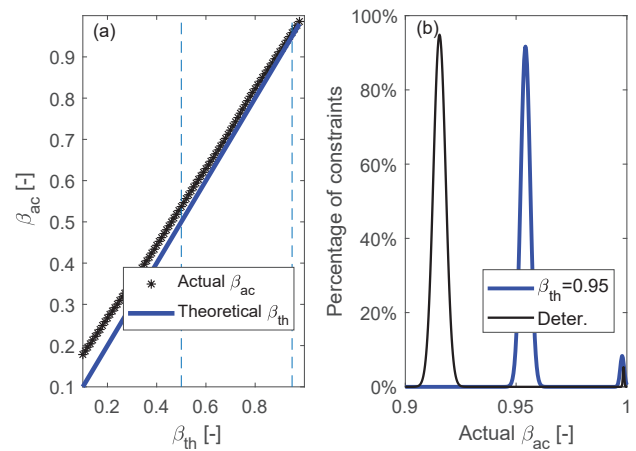


Fig. 8. CC validation for the case of daily rebound: (a) Imposed β_{th} and achieved β_{ac} in CC method for price set $\Delta\lambda_t^{\alpha*}$; (b) Probability of β_{ac} for deterministic and stochastic case for price set $\Delta\lambda_t^{\alpha*}$.

F. Effect of the rebound constraint

In the simulation studies so far, we investigated the CC validation for the case of daily rebound. However, in reality, different consumers' categories can defer their loads for a shorter range of time, leading to a strict rebound. In order to quantify the effect of the rebound on the flexibility estimation, Table II reports the difference in flexibility obtained by the daily and strict rebound constraints. The values are calculated as the average amount of up-regulation flexibility (i.e., the amount of down-regulation will be the same, as we imposed perfect rebound in Eq.(5e)) provided during the day for the different price scenarios. It emerges that having a strict rebound reduced the flexibility provision by 35%.

In Fig. 9, the CC validation is repeated for the case of strict rebound.

TABLE II
ANALYSIS OF THE FLEXIBILITY PROVIDED DURING THE DAY,
CONSIDERING DIFFERENT REBOUND EFFECTS AND $\beta_{th}=0.95$.

Study case	Up- (down-) regulation (GW)
Daily rebound	0.374
Strict rebound	0.243
Difference	-35%

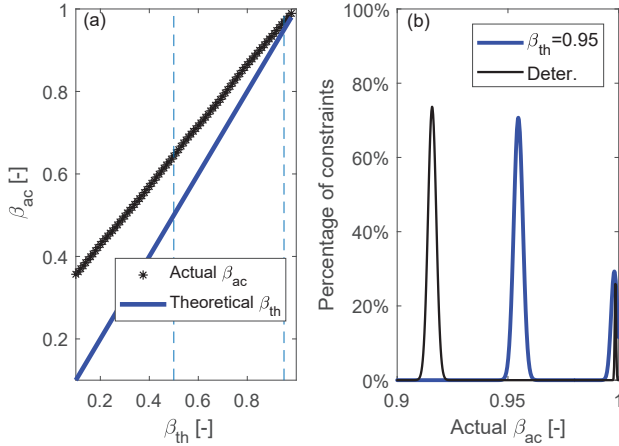


Fig. 9. CC validation for the case of strict rebound: (a) Imposed β_{th} and achieved β_{ac} in CC method for price set $\Delta\lambda_t^*$; (b) Probability of β_{ac} for deterministic and stochastic case for price set $\Delta\lambda_t^*$.

From Fig. 9(a), it can be seen that the actual confidence level is more conservative when dealing with a strict rebound by reaching β_{ac} of 0.96 for an imposed theoretical confidence level of 0.95. Also, Fig. 9(b) confirms the relevance of adopting CC, where the deterministic approach leads to a confidence level that is lower than 0.92 for 73% of the constraints. Such a result violates the requirement of the TSO, which imposed a β_{th} of 0.95.

VI. CONCLUSIONS

This paper offers a methodology to estimate the aggregated load flexibility of consumers given a certain price response function. It is formulated by considering the uncertainty in the consumers' willingness to react to the price signals. The proposed approach only requires aggregated historical consumption data to operate. In the proposed framework, the load flexibility at the TSO level is quantified. Time-varying prices are submitted by the system operator to the end-users at the edge of the grid to alter their consumption while minimising their operation cost locally. A nonlinear and stochastic consumers' price-response function is considered in this study. In order to quantify the risk in the amount of estimated demand flexibility, a CC formulation of the problem is developed and its applicability is proven by the simulation studies. This approach allows to estimate the flexibility that can be achieved under a certain confidence level. Actual load data from Elforbrugspanel in Denmark is used for simulation studies. The simulation results show that the choice of confidence level significantly affects the flexibility estimation. For a conservative confidence level (i.e., 0.95), the method

estimates a consumption change that is up to 7% of the total consumption. The quality of the CC solutions is also verified in two different ways. It is shown that the application of CC can provide a meaningful management of risk for the TSO, which is fundamental for AS provision. We finally evaluate the case of daily and strict rebound constraints, showing that a strict rebound effect limited the overall flexibility provision by 35%. The proposed approach can be used at the TSO level to quantify demand flexibility for day-ahead or real-time AS procurement. In our future work, we will investigate how to enhance our model to account for other uncertainties (such as uncertain delta prices) that the REU's EMS will most likely consider. Also, we will model $a_{t,j}^\alpha$ as a function of weather and type of day.

VII. ACKNOWLEDGEMENT

The work has been partly funded by CITIES project (DSF-1305-0027B) and the European Union's Horizon 2020 Smart-Net project (grant agreement No 691405).

The work by Juan M. Morales was supported in part by the European Research Council (ERC) under the EU Horizon 2020 research and innovation programme (grant agreement No. 755705) and in part by the Spanish Ministry of Economy, Industry and Competitiveness through project ENE2017-83775-P.

The authors are thankful to the Danish Council and EU for the support. Also, the authors would like to acknowledge Energinet and Dansk Energi for allowing the publication of part of the Elforbrugspanel data.

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